

The TRIUMF Cyclotron

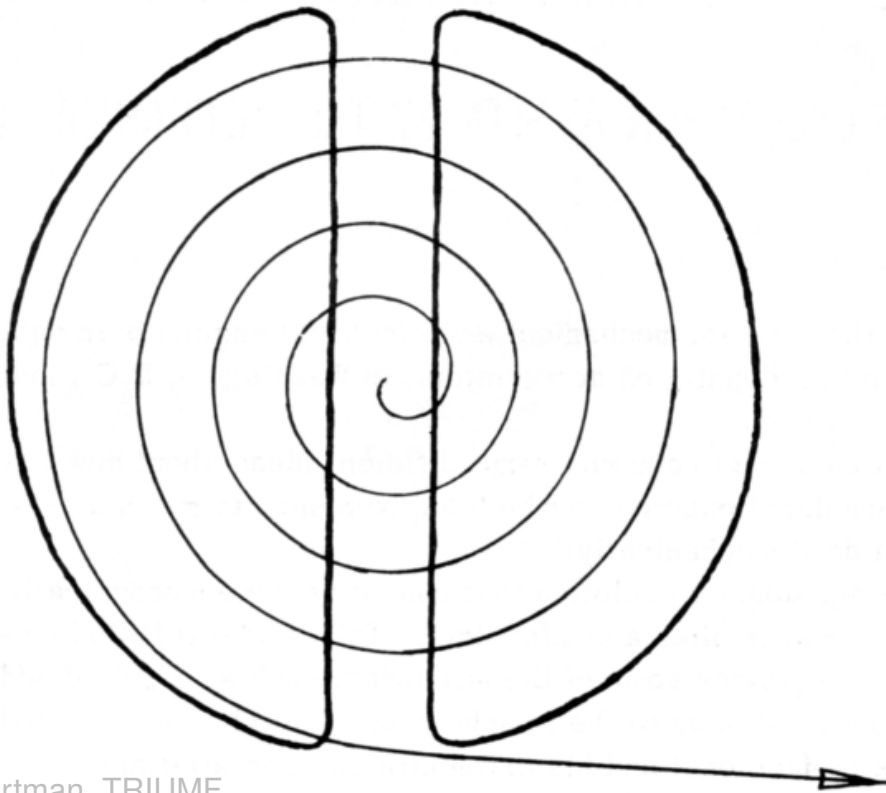


Rick Baartman, TRIUMF

April 18, 2013

Textbook Cyclotrons

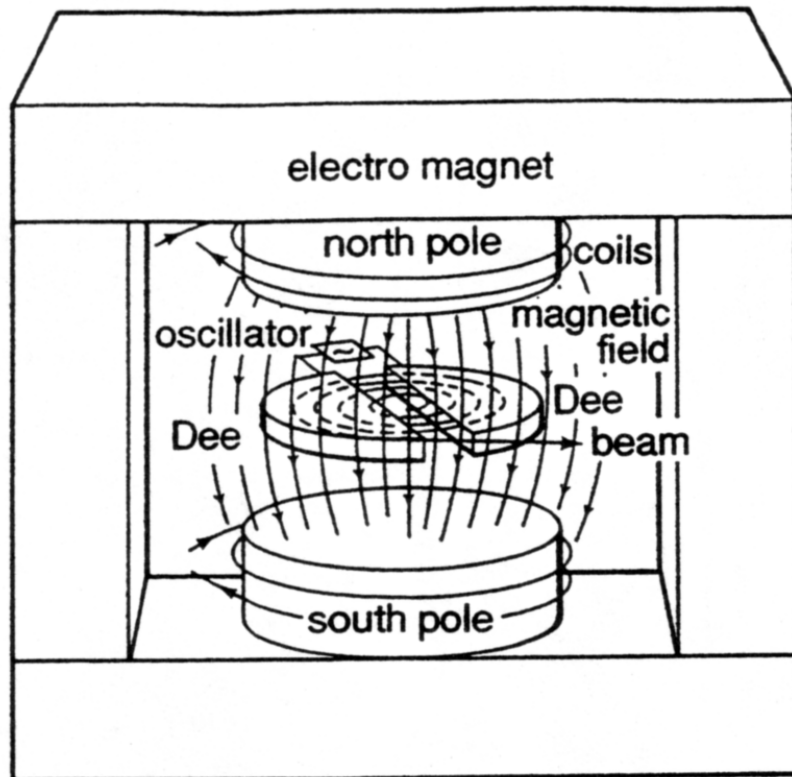
$$mv^2/r = qvB, \text{ so } m\omega_0 = qB, \text{ with } r = v/\omega_0$$



With B constant in time and uniform in space, as particles gain energy from the rf system, they stay in synchronism, but spiral outward in r .

1938 Cyclotron (Not like TRIUMF)

The natural decline of B with r actually helped. 1938: (R.R. Wilson) orbit theories developed, the effect is understood.



Flat field:

$\nu_z = 0$, $\nu_r = 1$. But a field which falls as r increases provides a restoring force toward the median plane.

No one thought of $B = B(\theta)$; only $B = B(r)$. Why?

Simple Cyclotron Orbits

Vertical
forces result from radial B :

$$F_z = qv B_r$$

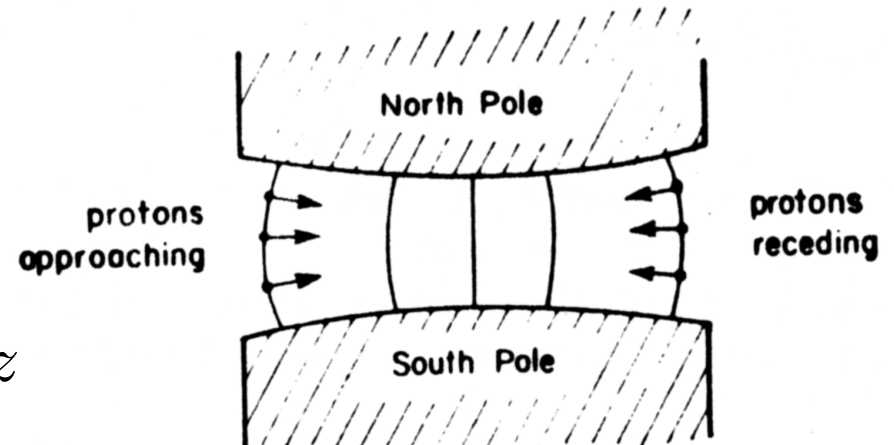
Taylor expand: $F_z \approx qv \frac{\partial B_r}{\partial z} z$

since $\nabla \times \vec{B} = \vec{0}$: $m\ddot{z} = qv \frac{\partial B_z}{\partial r} z$

This results in SHM of frequency

ω_z :

$$\omega_z^2 = - \frac{qv}{m} \frac{\partial B_z}{\partial r}$$



and tune $\nu_z = \omega_z/\omega_0$:

$$\nu_z^2 = -\frac{qv}{m\omega_0^2} \frac{\partial B_z}{\partial r} = -\frac{r}{B_z} \frac{\partial B_z}{\partial r} \equiv -\kappa$$

(κ is “field index”). Similarly, $\nu_r^2 = 1 + \kappa$. This sets the requirement $-1 < \kappa < 0$.

Field must fall monotonically with r or it blows up vertically.

Relativity

It turns out that for higher energies, the cyclotron resonance condition remains simple

$$m\gamma\omega_0 = qB, \text{ with } r = \beta c/\omega_0$$

That means

$$\kappa = \frac{\beta}{\gamma} \frac{d\gamma}{d\beta} = \beta^2\gamma^2$$

In other words, we cannot satisfy $-1 < \kappa < 0$: **Cannot have both vertical focusing and relativistic energy with this kind of cyclotron!**

Hans Bethe (1937): *... it seems useless to build cyclotrons of larger proportions than the existing ones... an accelerating chamber of 37 cm radius will suffice to produce deuterons of 11 MeV energy which is the highest possible...*

Such was Bethe's influence, that when a paper appeared in 1938, which appeared to resolve the problem, it was ignored for at least a decade. That paper was *The Paths of Ions in the Cyclotron* by L.H. Thomas.

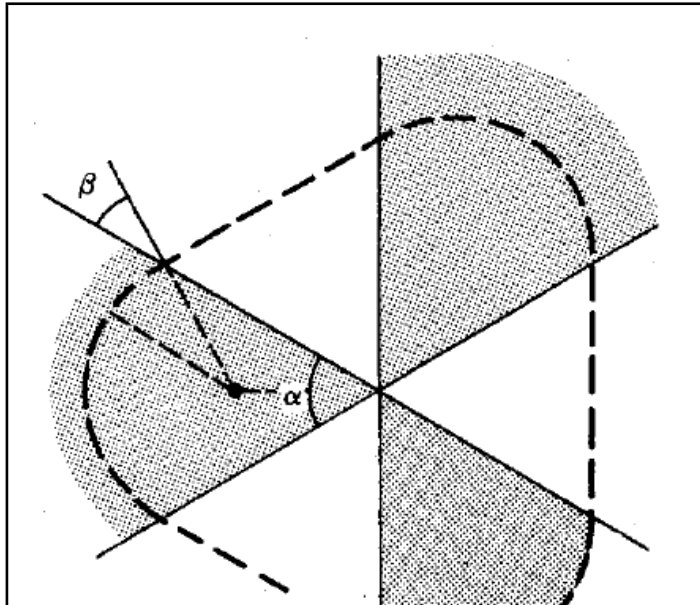
Frank Cole: *If you went to graduate school in the 1940s, this inequality $[-1 < \kappa < 0]$ was the end of the discussion of accelerator theory.*

This is NOT the kind of cyclotron we have at TRIUMF!

At TRIUMF, we have vertical focusing and $\gamma = 1.55$. How?

It is a different kind of cyclotron, invented by the MURA group in 1954.

Thomas focusing → FFAG



Separate the magnet into sector fields and drifts and you have edge focusing at every sector edge (Thomas, 1938). Replace drifts with negative fields, and get even more focusing (Ohkawa 1953; Symon, Kerst, Jones, Laslett, Terwilliger, 1956).

PHYSICAL REVIEW

VOLUME 103, NUMBER 6

SEPTEMBER 15, 1956

Fixed-Field Alternating-Gradient Particle Accelerators*

K. R. SYMON,[†] D. W. KERST,[‡] L. W. JONES,[§] L. J. LASLETT,^{||} AND K. M. TERWILLIGER[§]
Midwestern Universities Research Association

(Received June 6, 1956)

Fixed-Field Alternating-Gradient Particle Accelerators*

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Application of alternating-gradient focusing, however, can keep both modes of betatron oscillation stable even with the rapid radial change of magnetic field. Circular particle accelerators can be classified into four groups according to the type of guide field they use: fixed-field constant-gradient (conventional cyclotrons, synchrocyclotrons, and microtrons), pulsed-field constant-gradient (weak-focusing synchrotrons and betatrons), pulsed-field alternating-gradient (AG synchrotrons), and fixed-field alternating-gradient (FFAG synchrotrons, betatrons, and cyclotrons).

Two types of FFAG design appear the most practical. The radial-sector type² achieves AG focusing by having the fields in the successive focusing and defocusing magnets vary in the same way with radius but with alternating signs (or in certain cases alternating magnitudes). Since the orbit in the reverse field magnet bends away from the center, the machine is considerably larger than a conventional AG machine¹ of the same energy having an equal-peak magnetic field. This serious disadvantage is largely overcome in the spiral-sector type³ in which the magnetic field consists of a radially increasing azimuthally independent field on which is superimposed a radially increasing azimuthally periodic field. The ridges (maxima) and troughs (minima) of the periodic field spiral outward at a small angle to the orbit. The radial separation between ridges is small compared to the radial aperture. The particle, crossing the field ridges at a small angle, experiences alternating-gradient focusing. Since the fields need not be reversed anywhere, the circumference of this machine can be comparable to that of an equivalent conventional AG machine.

15. FFAG Cyclotrons

To make semirelativistic particles revolve in a cyclotron at constant frequency and in orbits that are approximately circles, it is necessary to have the average magnetic field increase with radius. In order to avoid the resultant axial defocusing, alternating-gradient focusing may be employed. There are a number of possible magnetic field configurations for such a fixed-field alternating-gradient cyclotron. The first such cyclotron was proposed by Thomas.⁵ The Thomas cyclotron is essentially a radial-sector FFAG machine having three or more sectors with a roughly sinusoidal field flutter. Thomas showed that such a machine has stable orbits for energies up to a limit depending upon the number of sectors. A considerable amount of experimental and theoretical work on the Thomas

1858

SYMON, KERST, JONES, LA

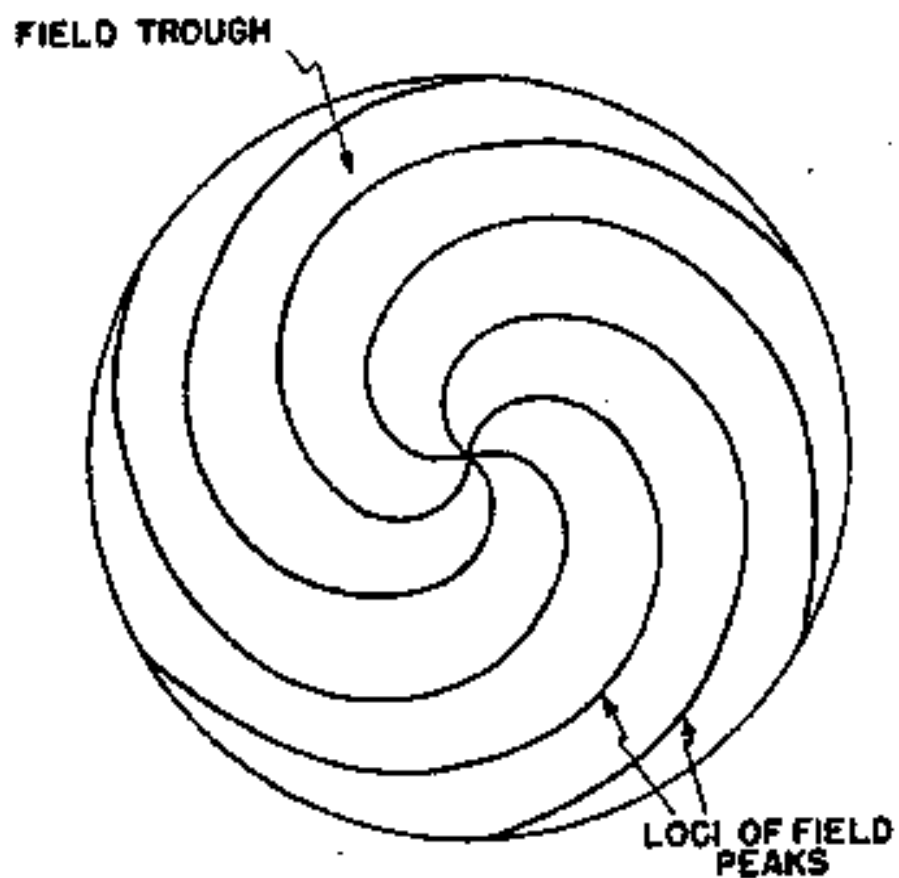
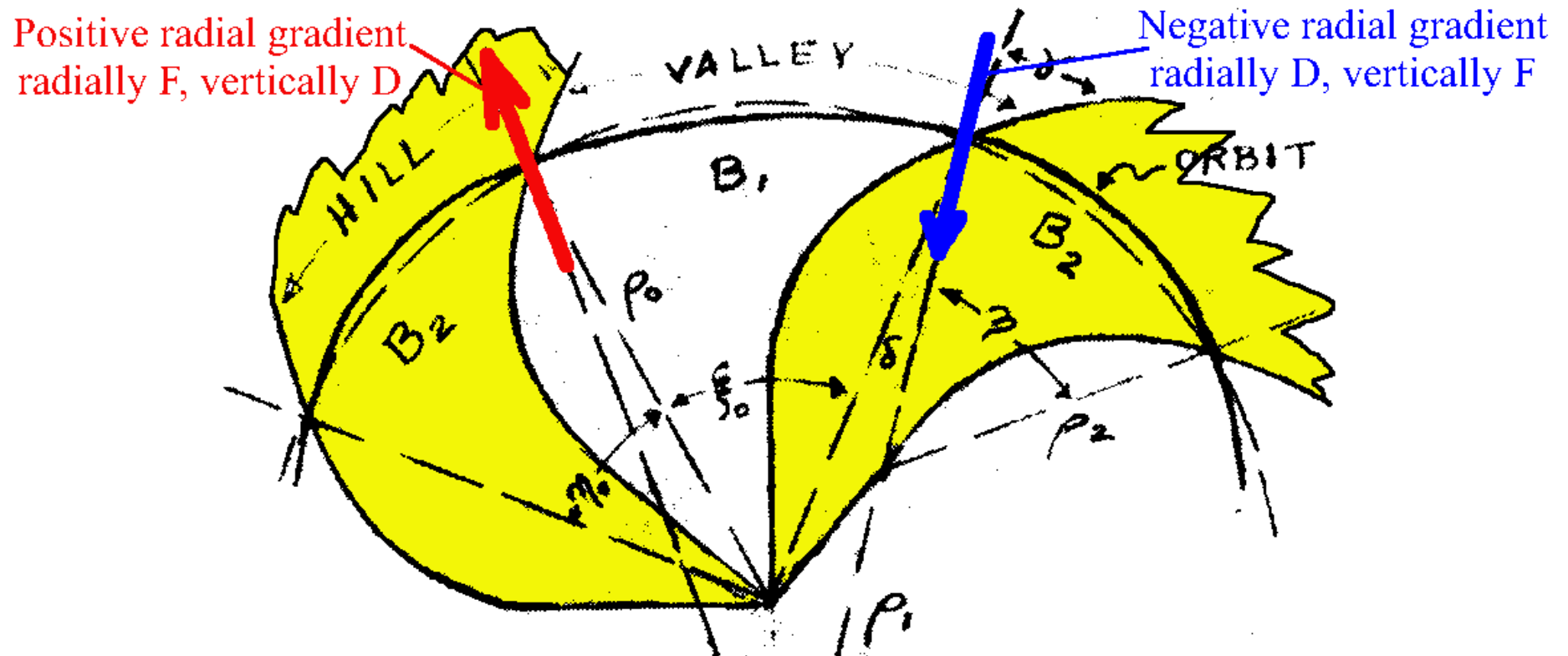


FIG. 24. Plan view of ridges in a 6-sector spiral-sector cyclotron.

Spiral focusing



It's strong focusing with a radially extended gap.

Isochronism

Orbit length L is given by speed and orbit period T :

$$L = \oint ds = \oint \rho d\theta = \beta c T.$$

The local curvature $\rho = \rho(s)$ can vary and for reversed-field bends (Ohkawa, 1953) even changes sign. (Along an orbit, $ds = \rho d\theta > 0$ so $d\theta$ is also negative in reversed-field bends.) Of course on one orbit, we always have $\oint d\theta = 2\pi$. What is the magnetic field averaged over the orbit?

$$\overline{B} = \frac{\oint B ds}{\oint ds} = \frac{\oint B \rho d\theta}{\beta c T}.$$

But B_ρ is constant and in fact is $\beta\gamma mc/q$. Therefore

$$\overline{B} = \frac{2\pi m}{T q} \gamma \equiv B_c \gamma = \frac{B_c}{\sqrt{1 - \beta^2}}.$$

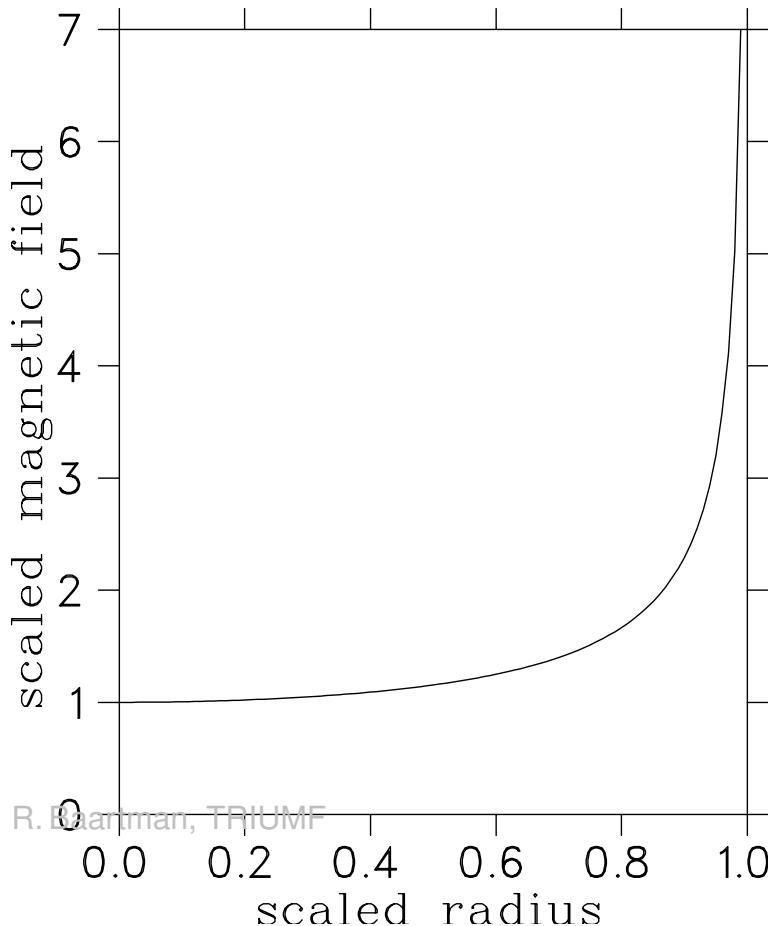
Remember,

β is related to the orbit length:

$\beta = L/(cT) = 2\pi R/(cT) \equiv R/R_\infty$. So

$$\overline{B} = \frac{B_c}{\sqrt{1 - (R/R_\infty)^2}}.$$

Of course, this means the field index is $\kappa = \frac{R}{B} \frac{dB}{dR} = \frac{\beta}{\gamma} \frac{d\gamma}{d\beta} = \beta^2 \gamma^2 \neq \text{constant}$.



Tunes

$$\nu_r^2 = 1 + \kappa, \text{ and } \nu_z^2 = -\kappa + F^2(1 + 2 \tan^2 \xi)$$

These expressions were originally derived by Symon, Kerst, Jones, Laslett, Terwilliger in the 1956 Phys. Rev. paper.

For **isochronous** machines, we therefore have

$$\nu_r = \gamma, \text{ and } \nu_z^2 = -\beta^2\gamma^2 + F^2(1 + 2 \tan^2 \xi)$$

Recap: FFAG cyclotrons

- Orbit length $\propto \beta$ (obvious for isochronism).
- B averaged on orbit $\propto \gamma$.
- Radial tune = γ .
- Vertical tune depends upon size of relative variation of $B(\theta)$, and on spiral angle.



| Energy | \bar{B} | R | $\beta\gamma$ | ξ | $1 + 2 \tan^2 \xi$ | F^2 |
|---------|-----------|---------|---------------|-------|--------------------|-------|
| 100 MeV | 0.335T | 175 in. | 0.47 | 0° | 1.0 | 0.30 |
| 250 MeV | 0.383T | 251 in. | 0.78 | 47° | 3.3 | 0.20 |
| 505 MeV | 0.466T | 311 in. | 1.17 | 72° | 20.0 | 0.07 |

TRIUMF Details:

Magnet: 4,000 tons

RF volts per turn = 0.4 MV.

Number of turns to 500 MeV = 1250.

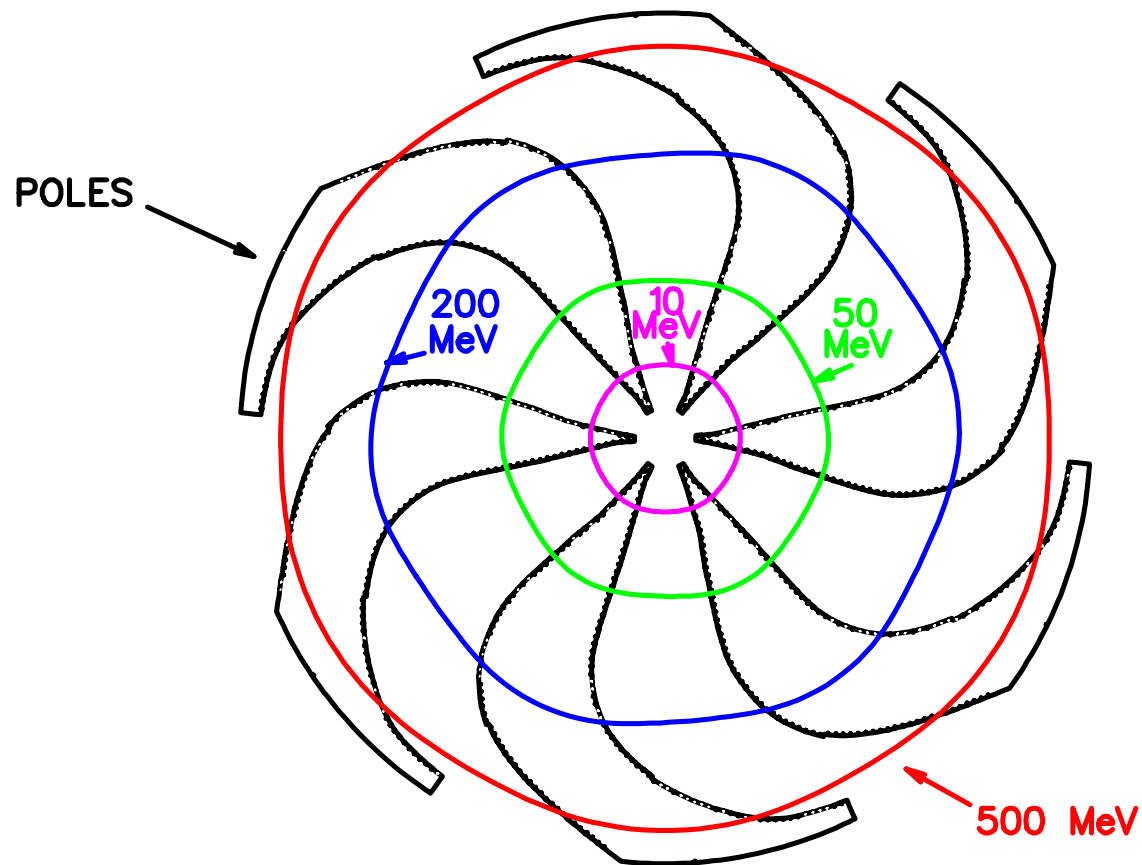
RF harmonic = 5: A magnetic field error of 1:12,500 results in a phase slip of 180° . This means magnetic field tolerance is a few ppm.

Injection energy is 0.3 MeV. That's a momentum range of a factor of 40.

Peak Intensity achieved: 0.42 mA. This would be 0.2 MW at full duty cycle.

PSI cyclotron has reached 2.4 mA at 590 MeV, 1.4 MW. The reason is that they have higher injection energy, stronger vertical focusing at injection.

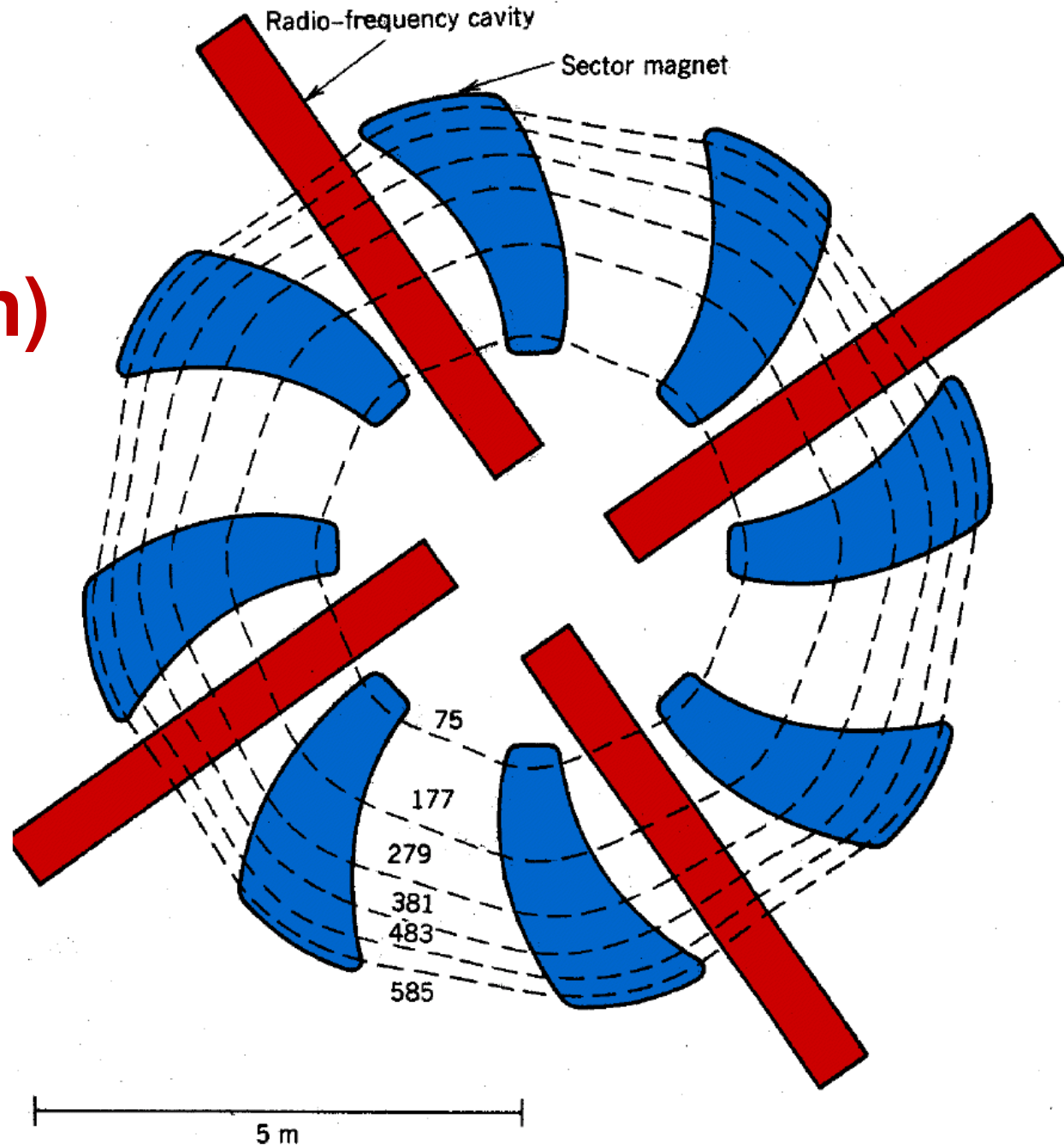
TRIUMF'S POLES AND EQUILIBRIUM ORBITS



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PSI cyclotron (for comparison)

Outer orbit
is 4.5m
compared
with
TRIUMF's
7.6m.



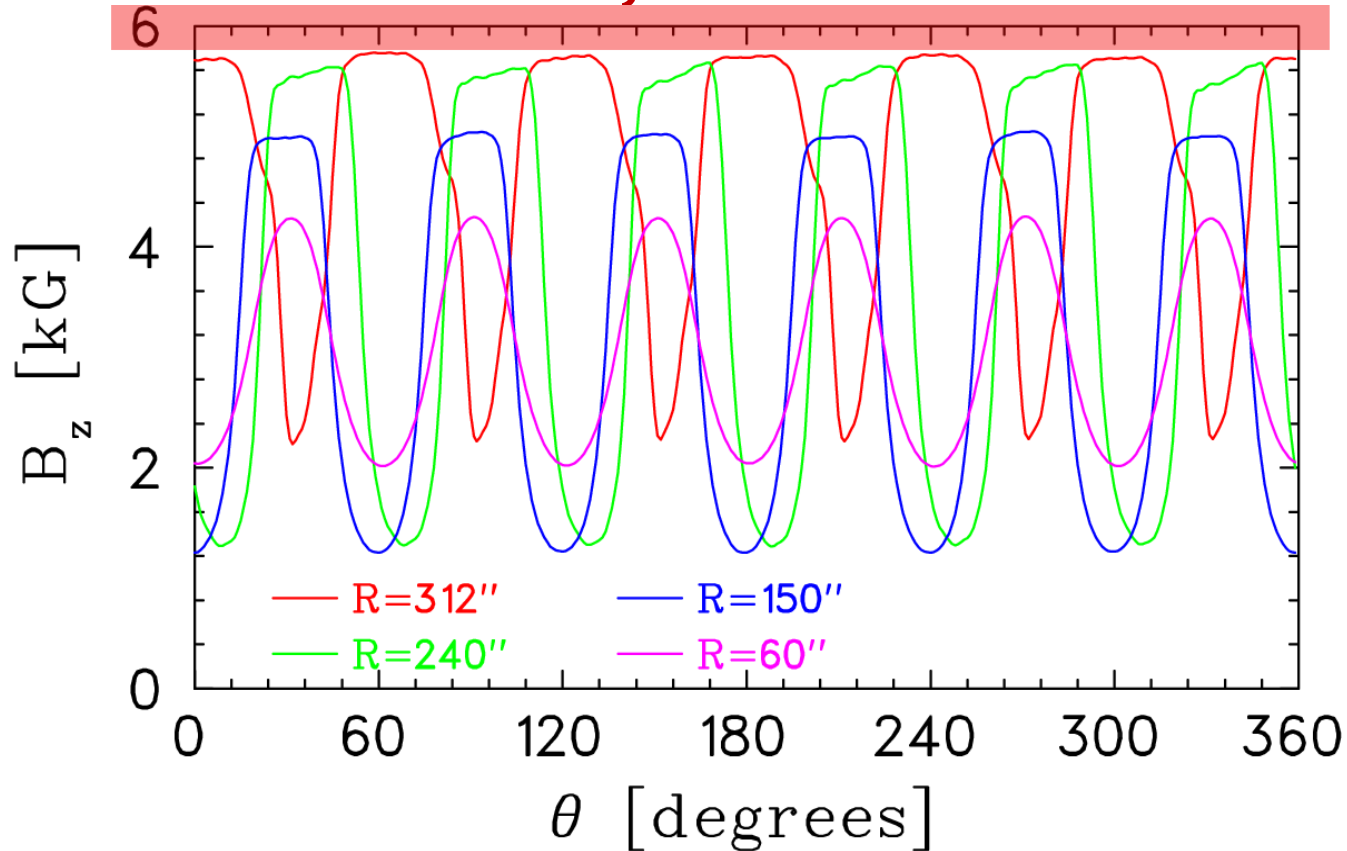
Why does TRIUMF cyclotron look so different?

Marriage of 2 ideas: FFAG and H^- ions.

But at 520 MeV, B cannot exceed 0.58 Tesla, or **Lorentz stripping**. This drastically reduces Flutter $\frac{\delta B}{B} \Big|_{\text{rms}}$. So must compensate with large spiral angle.

Stripping is about 5% at 500 MeV, 2% at 480 MeV.

At 500 MeV, cannot exceed 0.58 Tesla

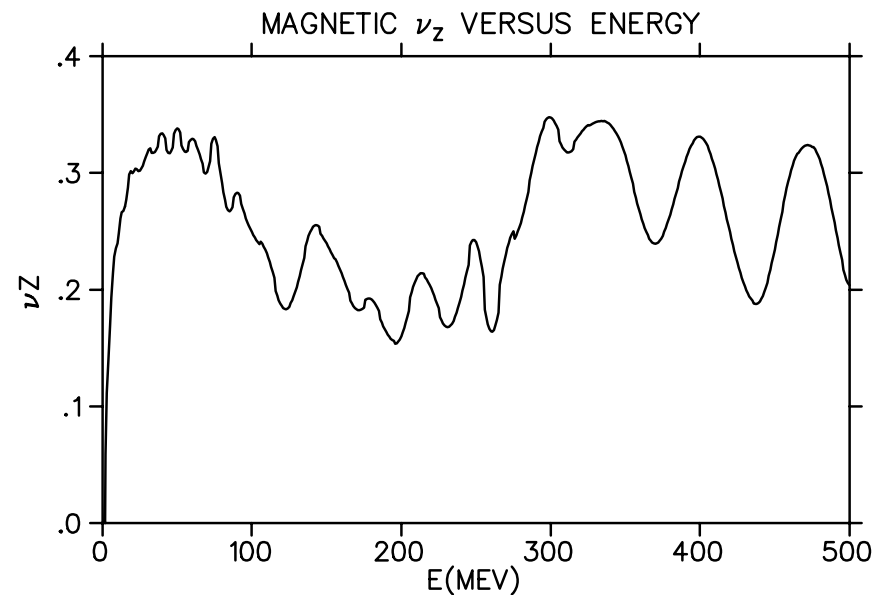
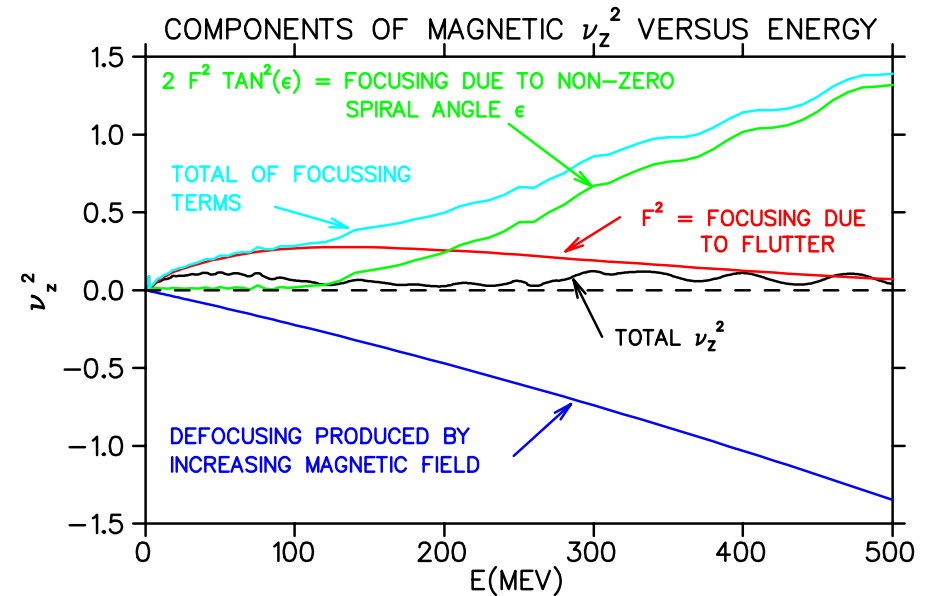


TRIUMF:

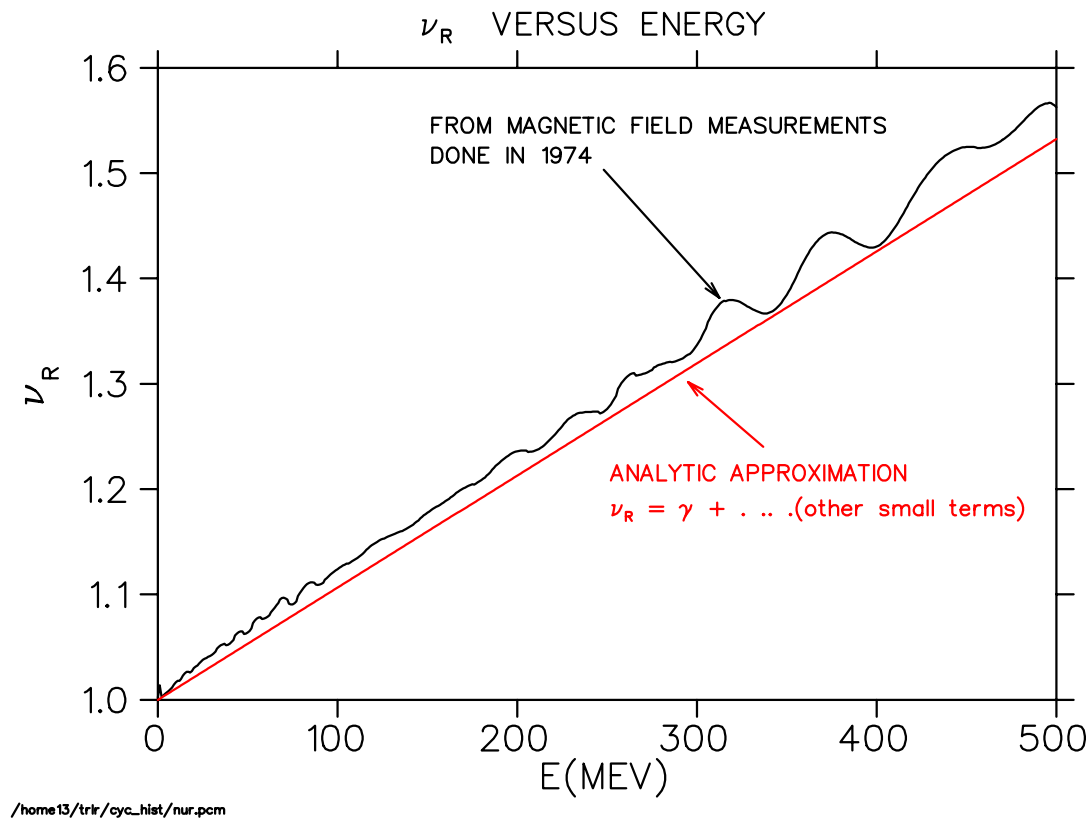
$\hat{B} = 5.8 \text{ kG}$, PSI: $\hat{B} = 20 \text{ kG}$. Spiral angle is 72° for TRIUMF, 35° for PSI.

Vertical focusing in TRIUMF

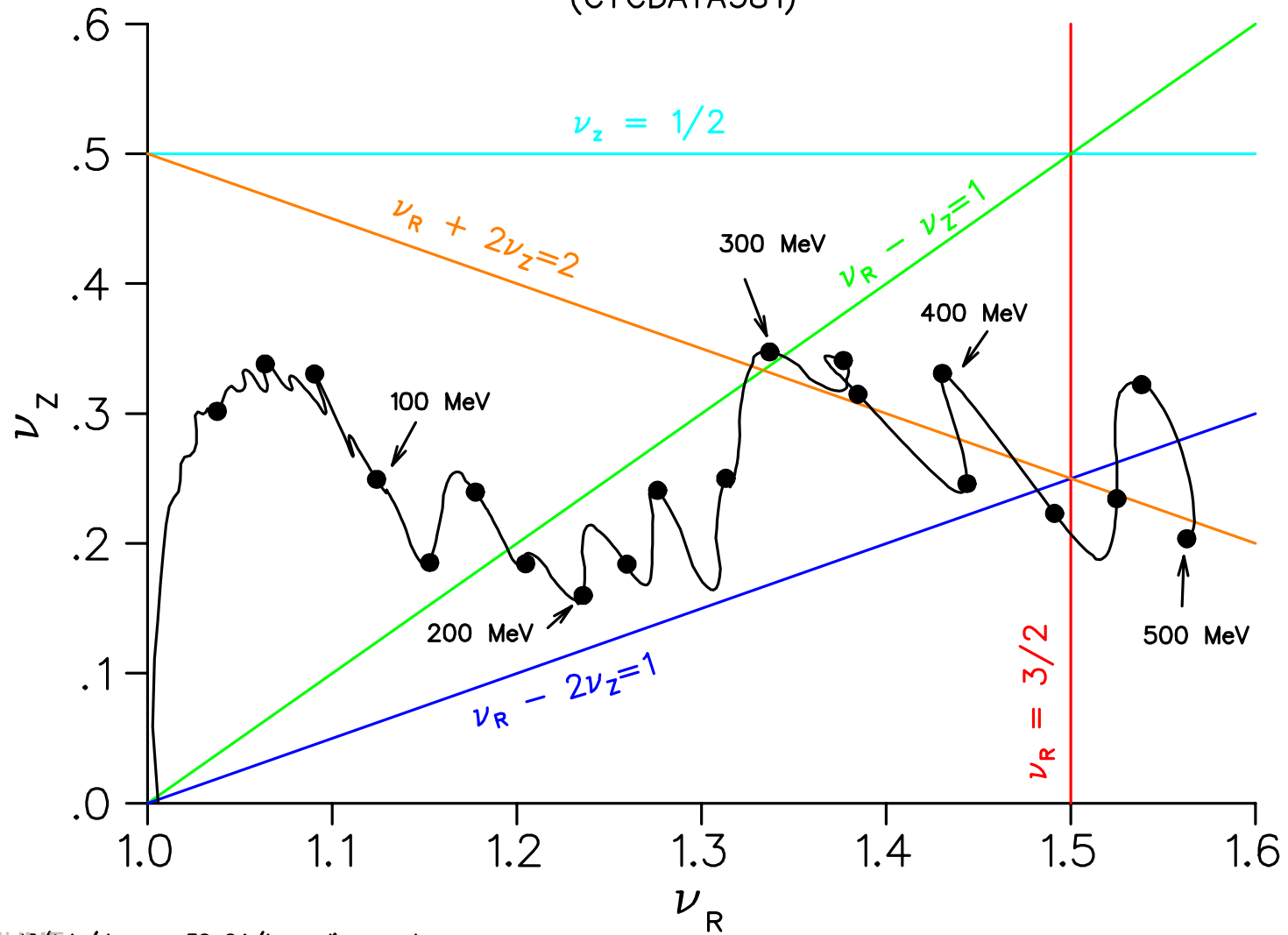
Peak B is low
because TRIUMF
accelerates
 H^- ; cannot
exceed 0.58 T.
Compare with PSI
(protons), where
peak field is 2 T.



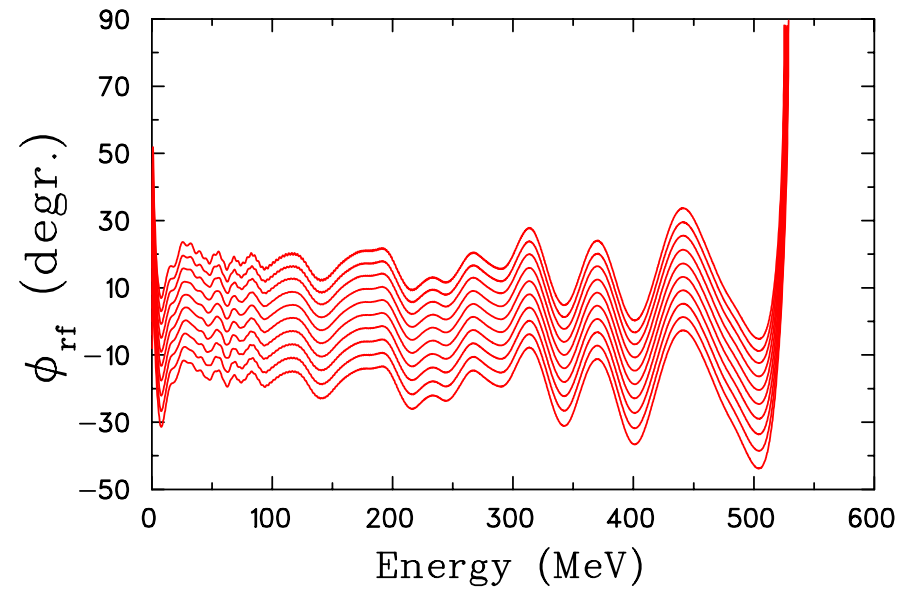
Radial Tune



TUNE DIAGRAM FOR TRIUMF (CYCDATA581)



Isochronism (Longitudinal phase space)



Isochronism (measured)

Take the previous graph, imagine that there is a mirror image at $\phi \rightarrow \phi + \pi$, and rotate it 90° .

Here is a longitudinal trajectory as measured by time-of-flight (Craddock et al, 1977 PAC).

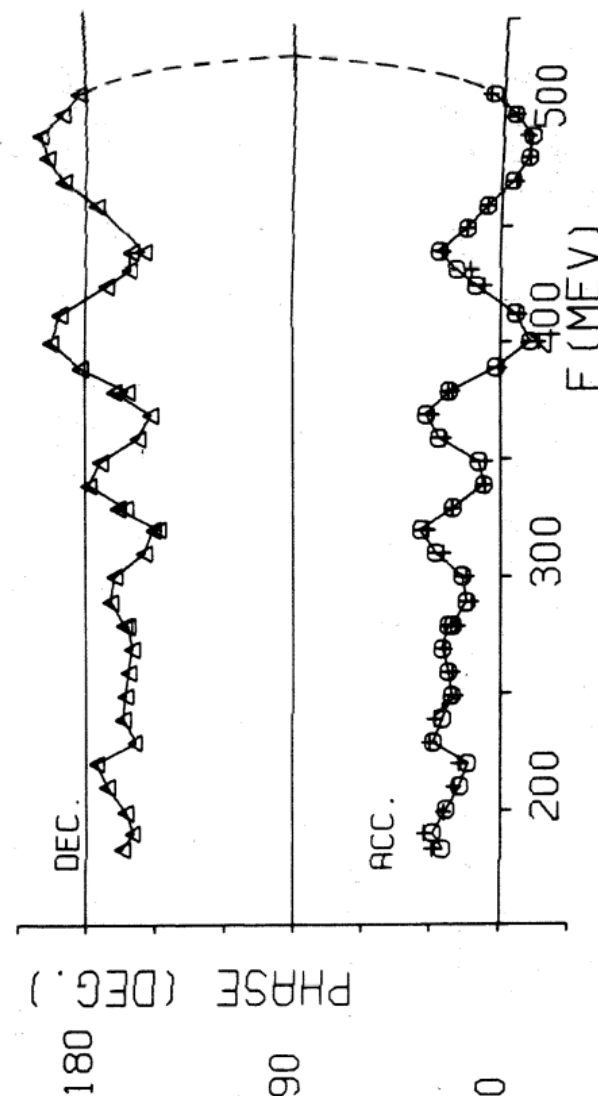
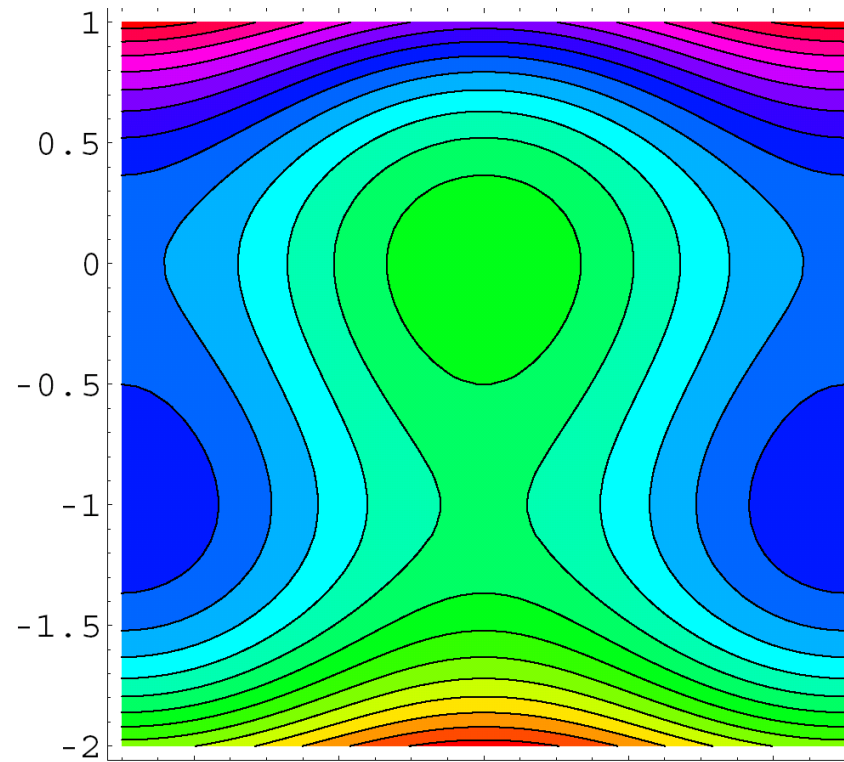


Fig. 2. Phase histories of accelerating and decelerating beams, obtained by timing an external beam.

Same sort of thing as EMMA:



TRIUMF cyclotron is:

- Fixed Field
- Alternating Gradient
- Non-scaling

So is EMMA the “World’s First Non-Scaling FFAG”?

THPP004

Proceedings of EPAC08, Genoa, Italy

EMMA – THE WORLD’S FIRST NON-SCALING FFAG

R. Edgecock, D. Kelliher, S. Machida, STFC/RAL, Didcot, UK

C. Beard, N. Bliss, J. Clarke, C. Hill, S. Jamison, A. Kalinin, K. Marinov, N. Marks, B. Martlew,
P. McIntosh, B. Muratori, H. Owen, Y. Saveliev, B. Shepherd, R. Smith, S. Smith, S. Tzenov,

C. White, E. Wooldridge, STFC/DL, Daresbury, UK

J.S. Berg, D. Trbojevic, BNL, Upton, New York, USA

M. Craddock, S. Koscielniak, TRIUMF, Vancouver, Canada

J. Crisp, C. Johnstone, FNAL, Illinois, USA

Y. Giboudot, Brunel University, UK

E. Keil, CERN, Geneva, Switzerland

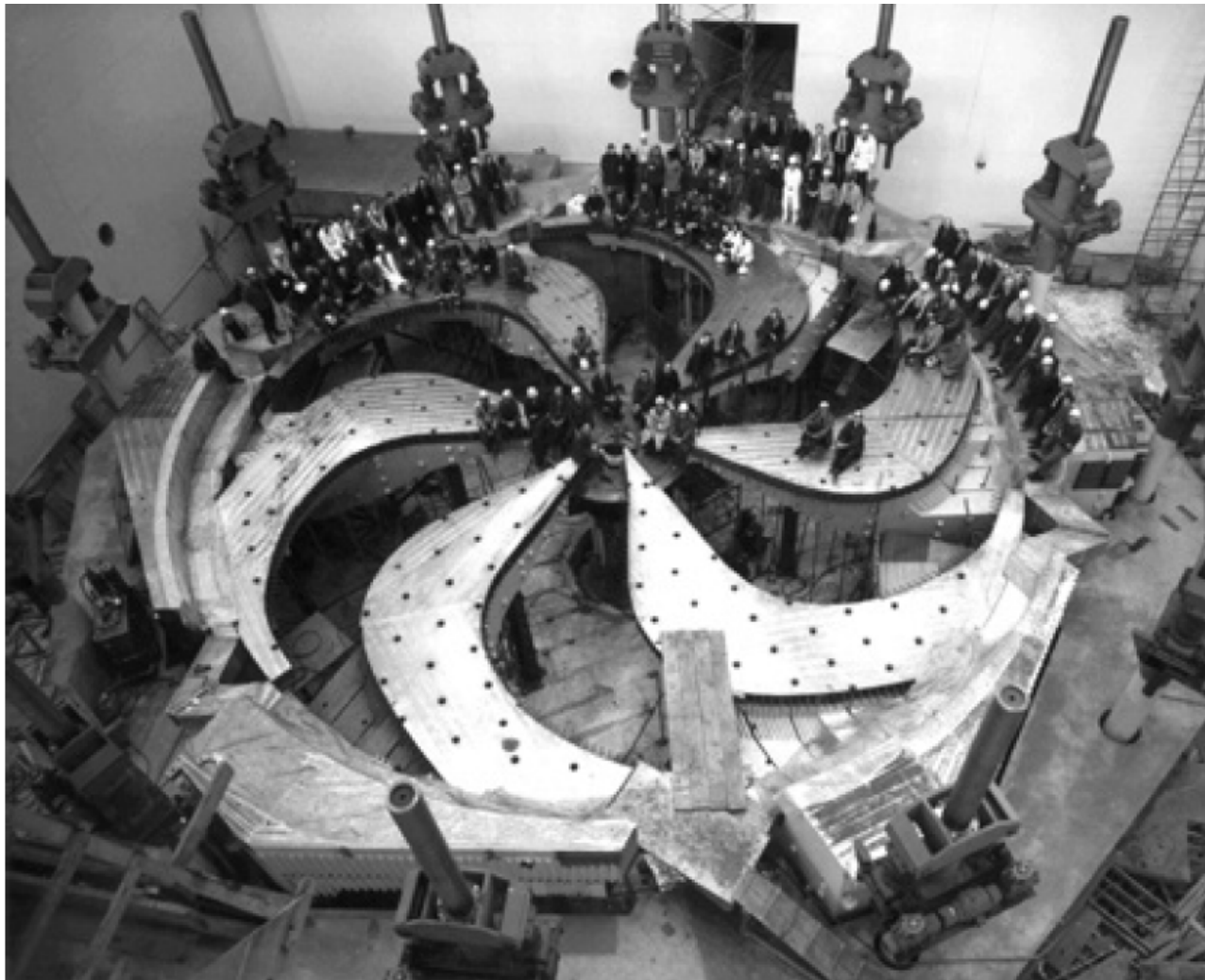
F. Méot, CEA & IN2P3, LPSC, France

T. Yokoi, Oxford University, UK

NO.

Let us doublecheck with one of the authors and definers of the FFAG genre:

Here is slide 27 taken directly out of Larry Jones' talk of May 2009 at APS meeting, Denver Colorado.



**The TRIUMF FFAG spiral sector cyclotron [520 MeV H^- ions]
at the University of British Columbia in Vancouver, Canada**

EMMA's Isochronism

EMMA is the first FFAG consisting entirely of quadrupoles. It is NOT isochronous in its energy range; only isochronous at ONE energy. It overcomes poor isochronism at other energies simply by brute force RF power. But it has an advantage: radial tune is only about $\gamma/2$.

Can one flatten the parabolic orbit time dependence on p ? Yes, but the radial tune would rise to γ (20 at injection, 40 at extraction). This would drive number of cells from 42 to > 80 .

Driving force behind the TRIUMF cyclotron was **J.R. Richardson** in mid-60s.

Very courageous to build such a large different machine with an incredible 72° spiral angle. But the 1956 paper did contain the vision(right).

1858

SYMON, KERST, JONES, LA

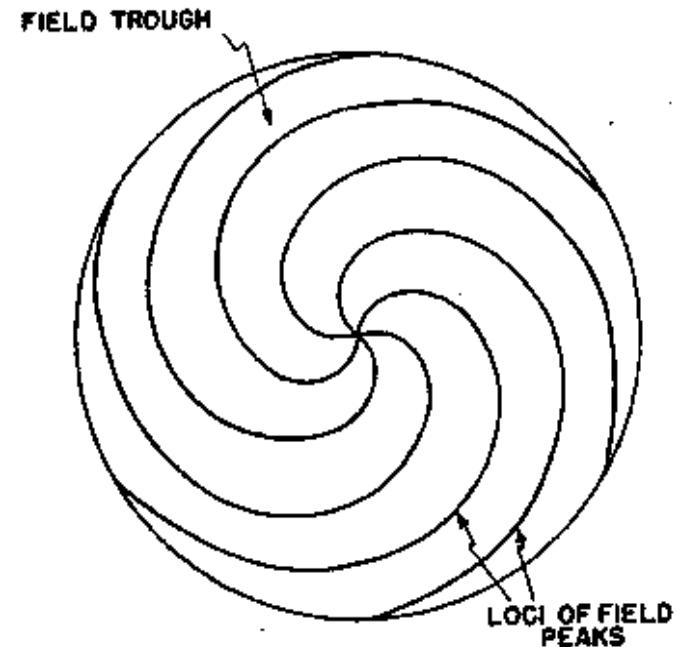


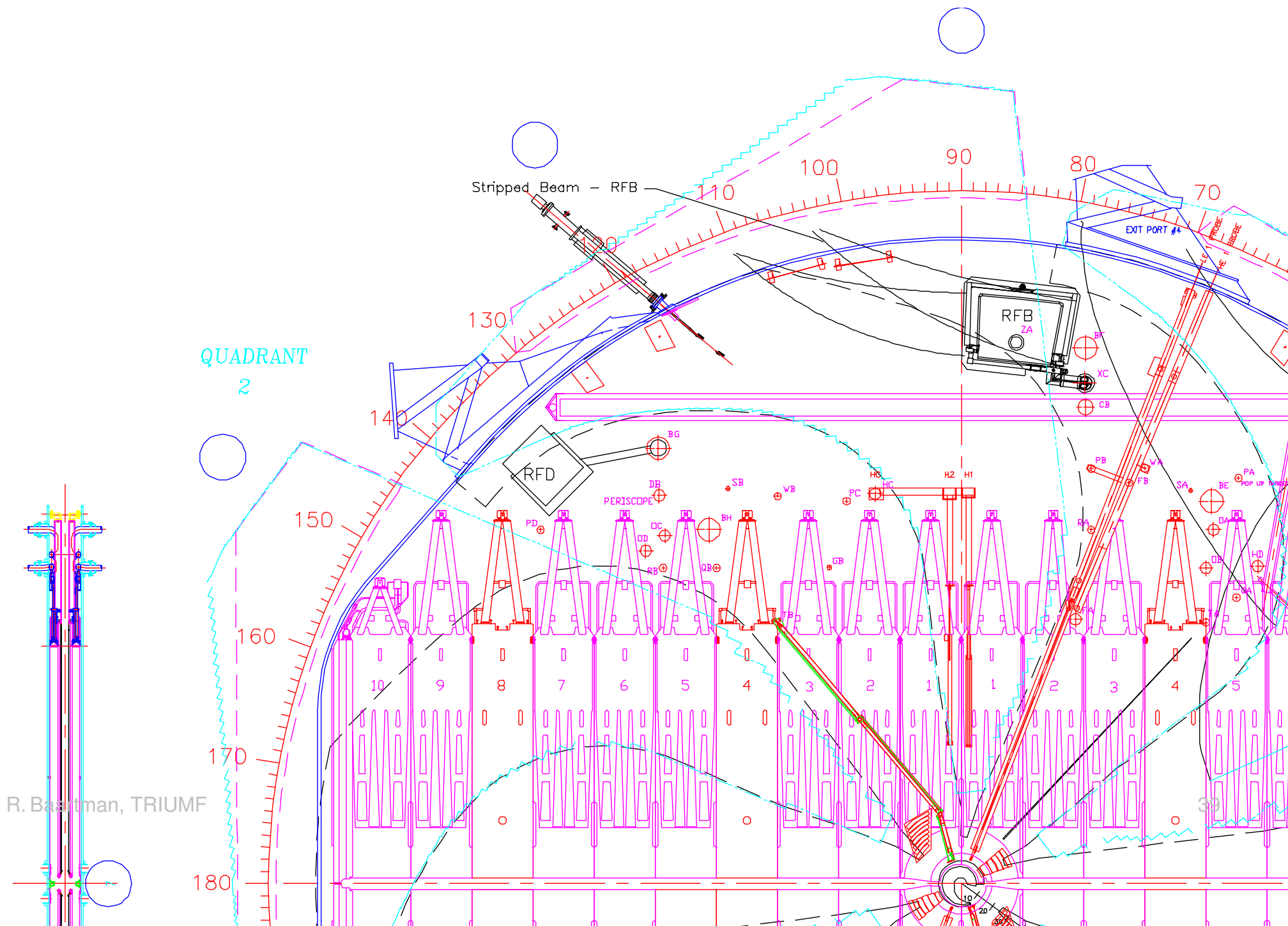
FIG. 24. Plan view of ridges in a 6-sector spiral-sector cyclotron.

Other implications of extreme spiral angle

Cannot fit the rf resonators between the sectors.

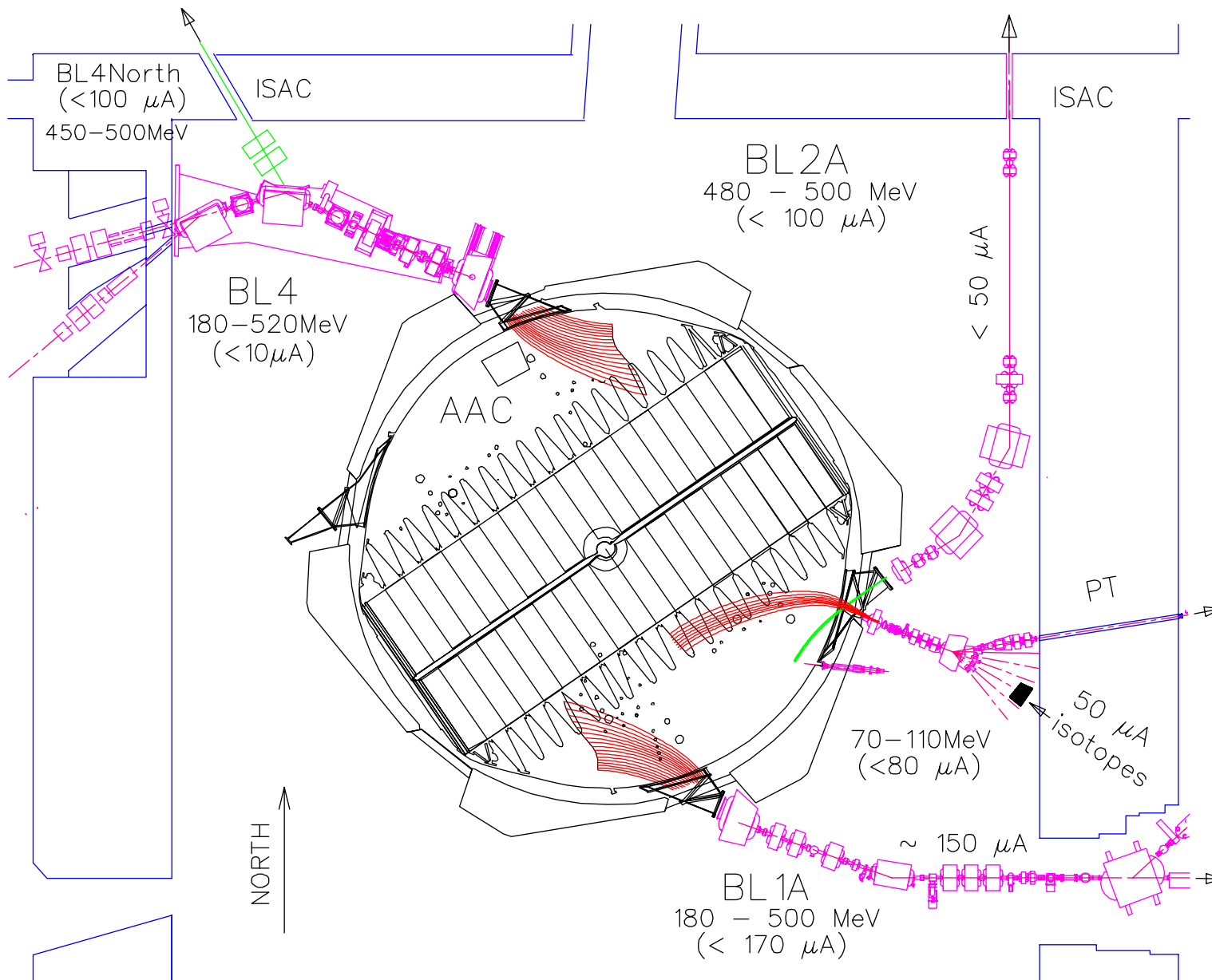
So it had to fit inside the magnet gap.

Inconvenient: gap must be large, rf system “on its side” has 3 metre cantilevered 1/4-wave resonators that oscillate (unavoidably; water cooling turbulence) at 5 Hz.



But advantages are Huge

- Continuously variable extraction energy 60 MeV to 520 MeV.
- Simultaneous extraction feeding many beamlines, all at different or same energies.
- Insensitivity to oscillations in rf V (cheaper RF).
- No need for separate turns (cheaper RF).
- Simple low energy injector (also reduces cost).



R. Baartman, TRIUMF

LAYOUT OF TRIUMF CYCLOTRON SHOWING EXTRACTED BEAMS

Non-intuitive features of H^- Extraction:

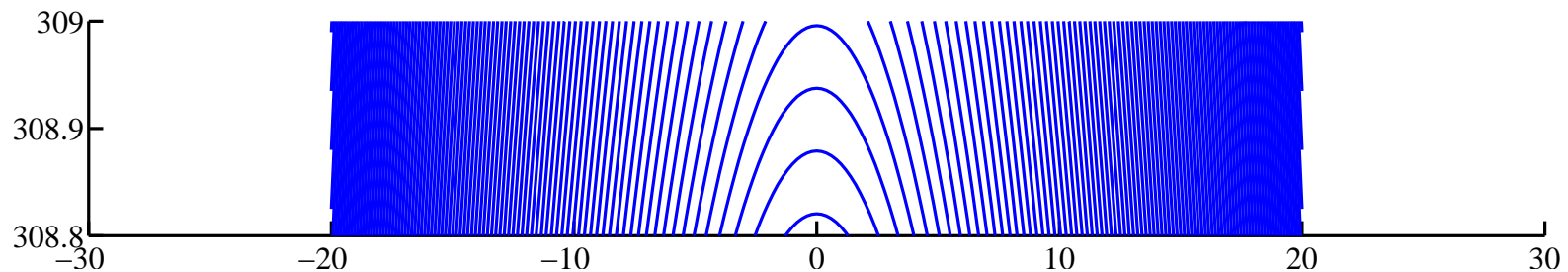
Phase acceptance

Extremely broad RF phase acceptance because we do not care how many turns it takes to get to extraction radius.

Example: “On crest” particle (Remember: It stays on crest as in an electron linac) takes 1250 turns to get to 500 MeV from 400 kV per turn. A particle displaced from crest by 30° phase takes $1250 / \cos 30^\circ = 1443$ turns. No matter. Only extracted once it gets to exactly correct energy.

Here's a section of turns, $\Delta E = 0.36$ MV per turn, phase width of $\Delta\phi = 40^\circ$.

To keep turns separate, would need $\Delta\phi < 5^\circ$. Even this would not work, as the betatron turn width is itself already larger than the separation. Radius gain per turn = 1.4 mm, $2\sqrt{\epsilon\beta_x} = 4$ mm.



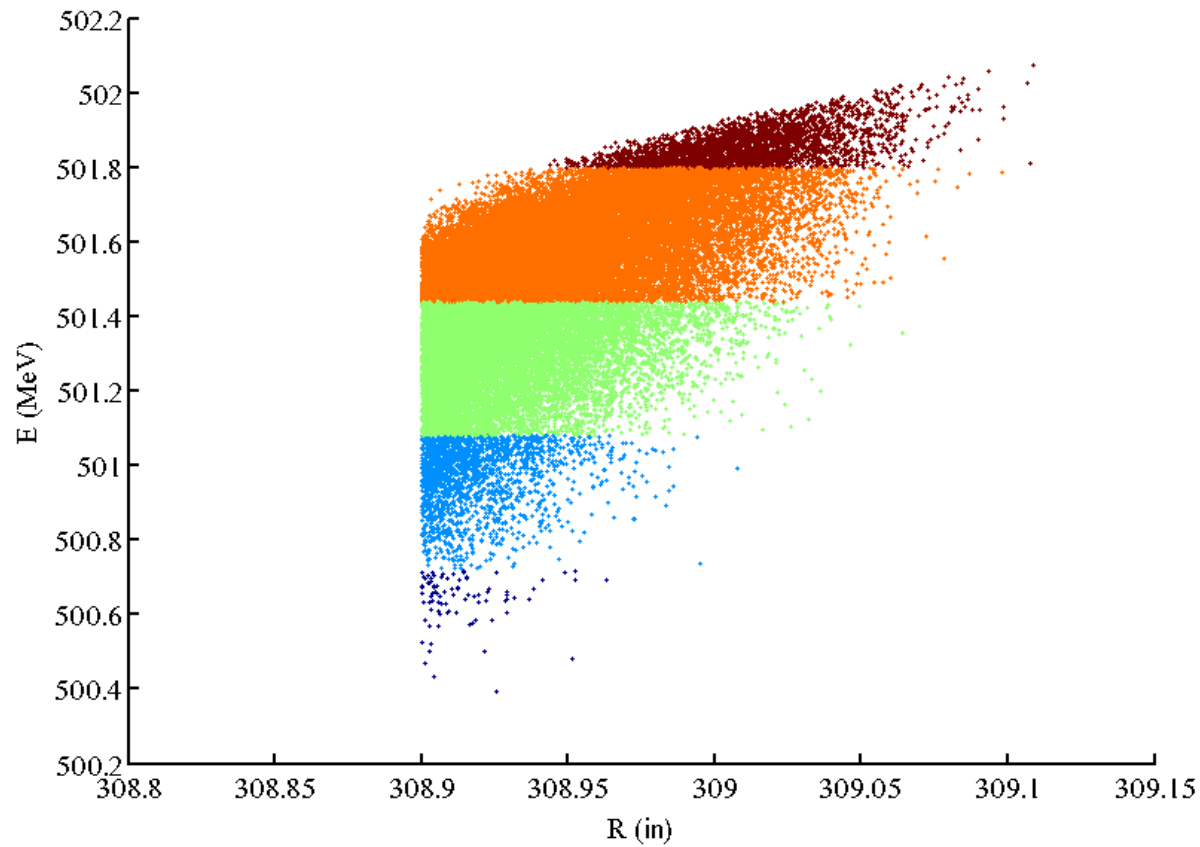
Plotted as $R/1$ inch vs. $\phi/1^\circ$.

This also means pulse rise time is SLOW ~ 400 RF periods or $17 \mu\text{s}$.

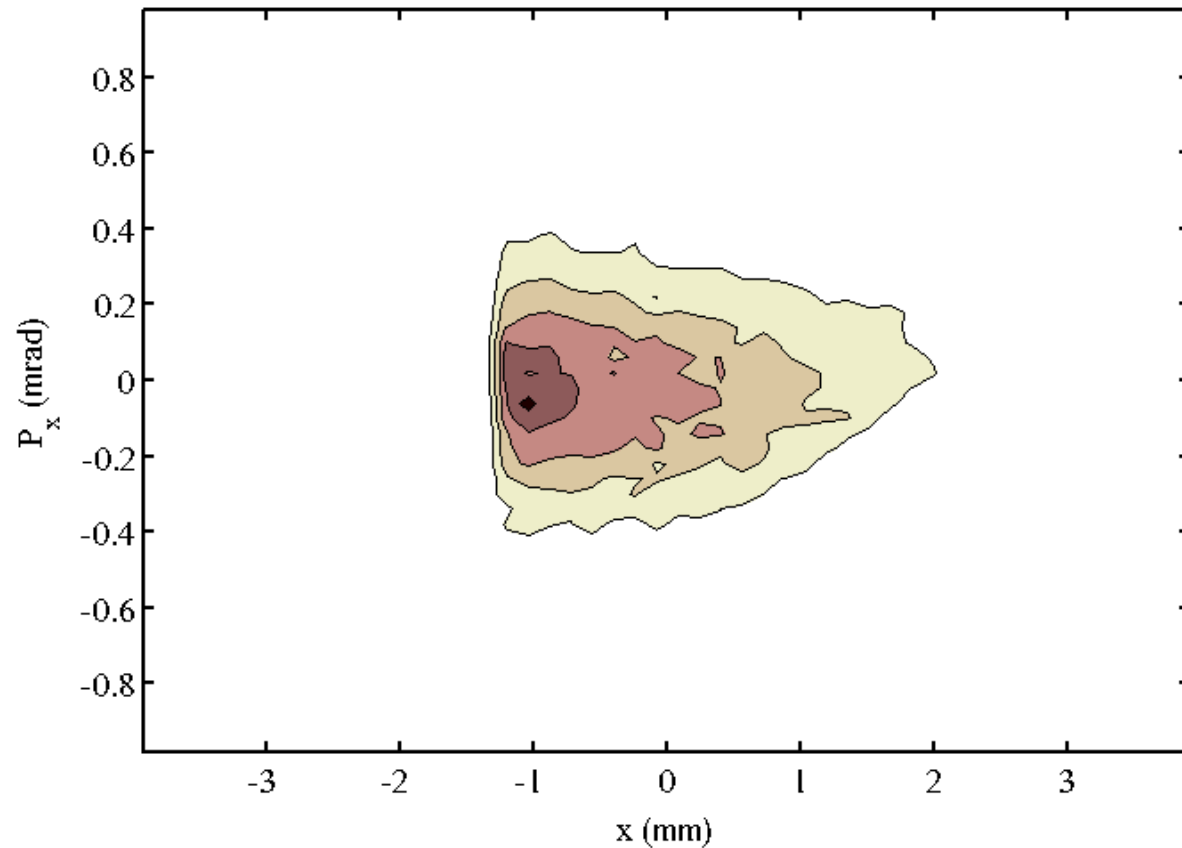
Non-intuitive features of H^- Extraction: Emittance

In spite of total mixing of many turns, radial emittance extracted is **smaller** than circulating emittance. Below is example of $\epsilon_r = 1 \mu\text{m}$ circulating, but extracted is 3/4 of this.

(Simulated) energy versus radius. Foil edge is at 308.9 inches.



(Simulated) extracted Horiz. phase space. $x = R - \bar{R}$ vs. P_x .
4rms emittance is $0.74 \mu\text{m}$.



Intensity Limit

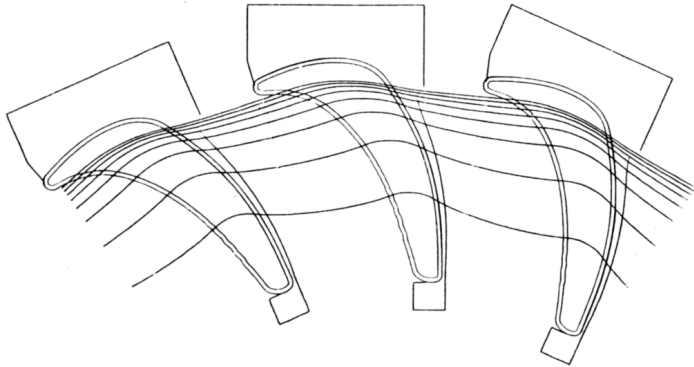
Activation limit: Can run continuously at $\sim 5 \mu\text{A}$ loss, one month cooldown before annual maintenance. E.g. $100 \mu\text{A}$ at 500 MeV, or $200 \mu\text{A}$ at 480 MeV, or up to space charge limit at 450 MeV.

Space Charge Limit: $420 \mu\text{A}$ observed, $\sim 500 \mu\text{A}$ calculated, depends strongly upon RF Voltage.

Foils: Pyrolytic graphite foils, 2 mg/cm^2 are good to $> 200 \mu\text{A}$. Heating is dominated by stripped electrons spiralling through again and again till extinguished. $\text{Power} = \text{Beam power} / 1836 * 2 = 100 \text{ Watts}$. Glows like a light bulb. At $120 \mu\text{A}$, foils last all year.

Can extend the idea?

Yes, but can only strip once. Thereafter need separated turn machines.

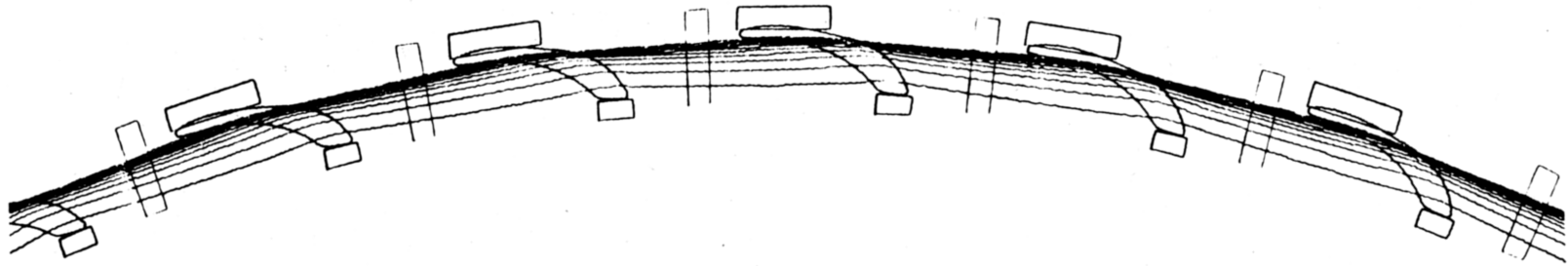


Sector design and orbits (at 0.5 GeV intervals)
for the 3.5 GeV cyclotron using single gullies.

For kaon factories, we designed high energy cyclotrons, but they were never built. Here are two: 3.5 GeV and 12 GeV (protons).

Nowadays
(now that superconducting rf has advanced), a 15 GeV proton

FFAG cyclotron would be much easier to build than a Muon FFAG (protons: $\beta\gamma\epsilon < 1 \mu\text{m}$). Multi-MW would be possible!



Five sectors from the 40 m radius 42 sector cyclotron showing magnets, RF cavities and proton orbits.

15 GeV. Needs about 50 MV per turn to get the turn separation.

$$\nu_r^2 = 1 + \kappa, \text{ and } \nu_z^2 = -\kappa + F^2(1 + 2 \tan^2 \xi), \text{ and } \kappa = \beta^2 \gamma^2$$

Therefore, it is necessary that $F^2(1 + 2 \tan^2 \xi) > \beta^2 \gamma^2$. The TRIUMF cyclotron is a demonstration that $(1 + 2 \tan^2 \xi)$ can be as large as 20. If F is only of order 1, we can already get $\beta\gamma > 4$. That's 3 GeV for protons! To get higher, F must be larger than 1, so reverse bend areas. This makes the rings larger, but this is needed anyways to get the turn separation at extraction.

Summary

TRIUMF cyclotron has operated reliably for almost 40 years.

It's a cost-effective route to > 100 kW beam power, > 500 MeV beam energy. It's continuous wave, not pulsed. Extremely bright ($\beta\gamma\epsilon_{\text{rms}} < 0.25 \mu\text{m}$).



Why can't scaling be isochronous?

$$p \propto B\rho \propto BR \propto R^{\kappa+1}$$

$$\text{Note: } R \propto p^{\frac{1}{\kappa+1}}.$$

Field index is κ . Take $p = \beta\gamma$, then $\beta = \frac{p}{\sqrt{1+p^2}}$, and:

$$T \propto \frac{R}{\beta} \propto \frac{\sqrt{1+p^2}}{p^{\frac{\kappa}{\kappa+1}}}$$

This works at $p \ll 1$ for $\kappa = 0$: Scaling FFAG is isochronous only at non-relativistic energy.

For $p \gg 1$, $T \propto p^{\frac{1}{\kappa+1}}$: almost works if κ is large enough.